UNIT- II RESONANCE and NETWORK THEOREMS

1. Explain series resonance and parallel resonance circuits.

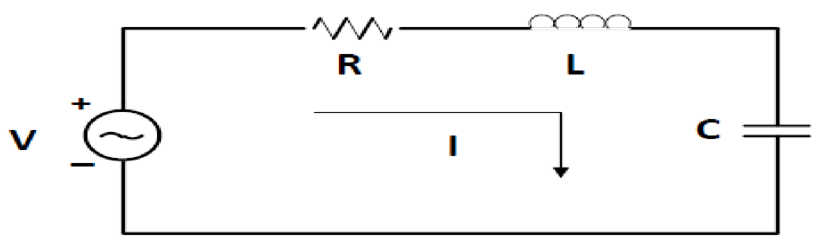
Any passive electric circuit will resonate if it has an inductor and capacitor.

Resonance is characterized by the input voltage and current being in phase. The driving point impedance (or admittance) is completely real when this condition exists.

SERIES RESONANCE

Consider the series RLC circuit shown below.

$V = V_M \angle 0$



The input impedance is given by:

$$Z = R + j(wL - \frac{1}{wC})$$

The magnitude of the circuit current is;

$$I = |\overline{I}| = \frac{V_m}{\sqrt{R^2 + (wL - \frac{1}{wC})^2}}$$

Resonance occurs when,

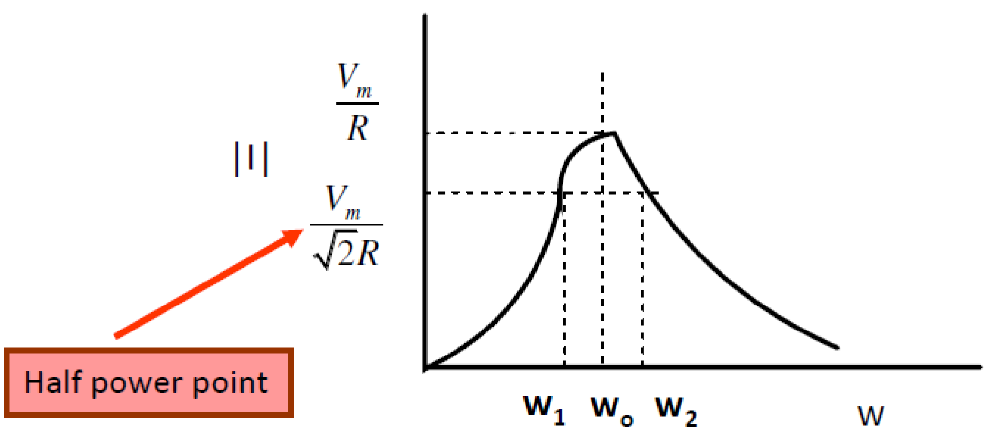
$$\boxed{wL = \frac{1}{wC}}$$

At resonance we designate w as w_o and write;

$$\boxed{w_o = \frac{1}{\sqrt{LC}}}$$

This is *an important equation* to remember. It applies to both series And parallel resonant circuits.

The magnitude of the current response for the series resonance circuit is as shown below.



Bandwidth:

$$\boxed{BW = w_{BW} = w_2 - w_1}$$

The <u>peak power</u> delivered to the circuit is;

$$P = \frac{V_m^{\ 2}}{R}$$

The so-called half-power is given when $I = \frac{V_m}{\sqrt{2}R}$.

We find the frequencies, w_1 and w_2, at which this half-power occurs by using;

$$\sqrt{2}R = \sqrt{R^2 + (wL - \frac{1}{wC})^2}$$

After some insightful algebra one will find two frequencies at which the previous equation is satisfied, they are:

$$w_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

and

$$w_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

The two half-power frequencies are related to the resonant frequency by

$$w_o = \sqrt{w_1 w_2}$$

The bandwidth of the series resonant circuit is given by;

$$\boxed{BW = w_b = w_2 - w_1 = \frac{R}{L}}$$

We define the Q (quality factor) of the circuit as;

$$\boxed{Q = \frac{w_o L}{R} = \frac{1}{w_o RC} = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}}$$

Using Q, we can write the bandwidth as;

$$\boxed{BW = \frac{w_o}{Q}}$$

These are all important relationships.

An Observation:

If Q > 10, one can safely use the approximation;

$$w_1 = w_o - \frac{BW}{2} \qquad and \qquad w_2 = w_o + \frac{BW}{2}$$

These are useful approximations.

By using $Q = w_o L/R$ in the equations for w_1 and w_2 we have;

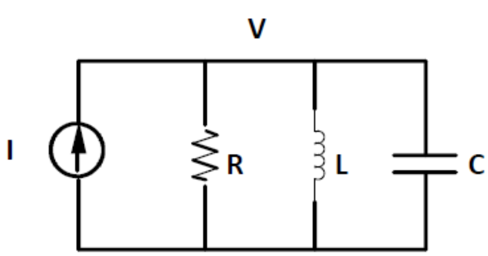
$$w_1 = w_o\left[\frac{-1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$$

and

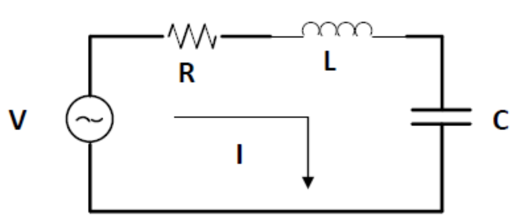
$$w_2 = w_o\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$$

PARALLEL RESONANCE

Consider the circuits shown below:



$$I = V\left[\frac{1}{R} + jwC + \frac{1}{jwL}\right]$$



$$V = I\left[R + jwL + \frac{1}{jwC}\right]$$

Parallel Resonance

$$w_o = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{w_o L}{R}$$

$$BW = (w_2 - w_1) = w_{BW} = \frac{R}{L}$$

$$w_1, w_2 = \left[\frac{\mp R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 +1} \right]$$

$w_1 w_2$

Series Resonance

$$w_o = \frac{1}{\sqrt{LC}}$$

$$Q = w_o RC$$

$$BW = w_{BW} = \frac{1}{RC}$$

$$w_1, w_2 = \left[\frac{\mp 1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 +1} \right]$$

A series RLC resonant circuit has a resonant frequency admittance of $2x10^{-2}$ S(mohs). The Q of the circuit is 50, and the resonant frequency is 10,000 rad/sec. Calculate the values of R, L, and C. Find the half-power frequencies and the bandwidth.

First, R = 1/G = 1/(0.02) = 50 ohms.

Second, from $Q = \frac{w_o L}{R}$, we solve for L, knowing Q, R, and w_o to find L = 0.25 H.

Third, we can use $$C = \frac{Q}{w_o R} = \frac{50}{10{,}000x50} = 100 \ \mu F$$

Fourth: We can use $$w_{BW} = \frac{w_o}{Q} = \frac{1x10^4}{50} = 200\, rad/\sec$$

and

Fifth: Use the approximations;

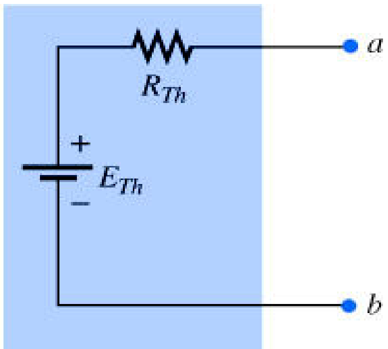
$w_1 = w_o - 0.5w_{BW} = 10{,}000 - 100 = 9{,}900$ rad/sec

$w_2 = w_o - 0.5w_{BW} = 10{,}000 + 100 = 10{,}100$ rad/sec

2. Network Theorems - Thevenin's,
Norton's,
Maximum Power Transfer,
Superposition,
Reciprocity,
Tellegen's,
Millman's
Compensation theorems
(for DC and AC excitations.)

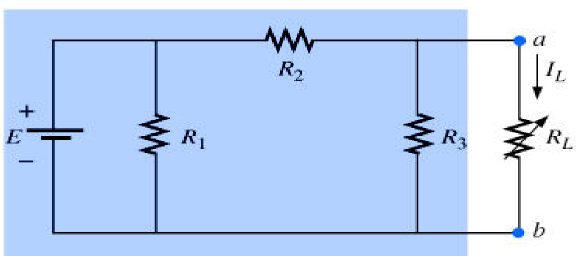
THEVININ'S THEOREM:

Any two-terminal dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.



Procedure to determine the proper values of R_{Th} and E_{th} :

1. Remove that portion of the network across which the Thévenin equivalent circuit is to be found. In the figure below, this requires that the load resistor R_L be temporarily removed from the network.



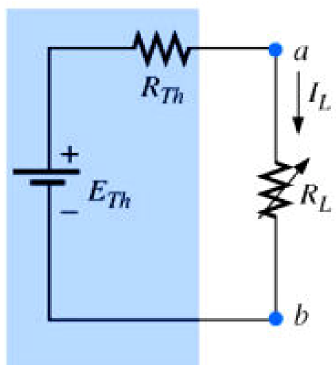
2. Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)

R_{Th}:

3. Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

E_{Th}:

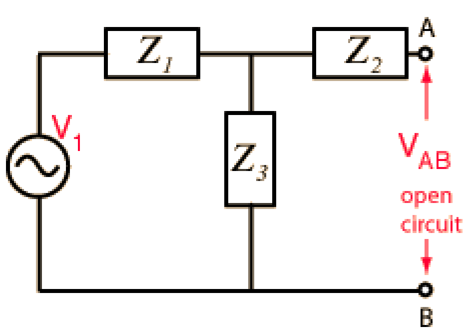
4. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that will lead to the most confusion and errors.In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.)



Conclusion:

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.This step is indicated by the placement of the resistor R_L between the terminals of the Thévenin equivalent circuit.

Numerical example for thevinin's theorem applied for ac circuit:



For impedances

Z_1 = [ ] + j[ ]

Z_2 = [ ] + j[ ]

Z_3 = [ ] + j[ ]

with supply voltage

V_1 = [ ] V,

NORTON'S THEOREM:

Norton's theorem states the following:

Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current and a parallel resistor.

The steps leading to the proper values of I_N and R_N.

1. Remove that portion of the network across which the Norton equivalent circuit is found.

2. Mark the terminals of the remaining two-terminal network.

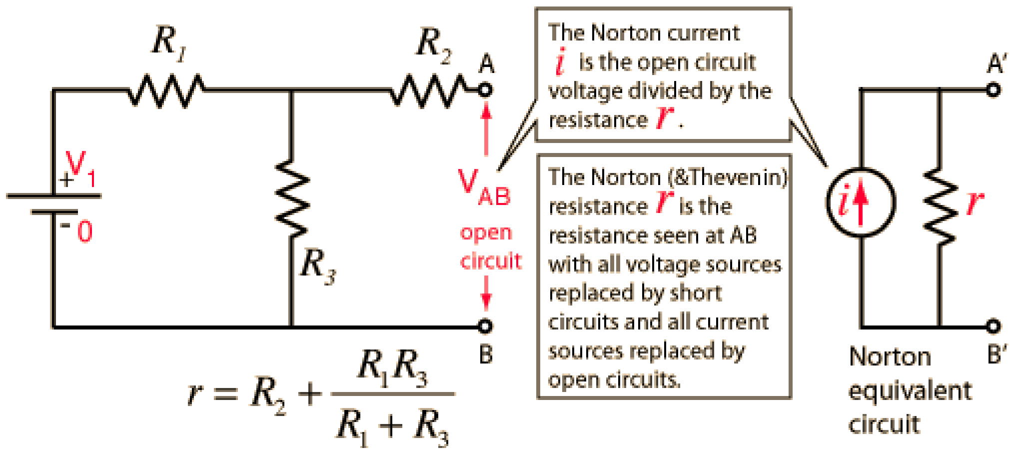
Finding R_N: Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)Since $R_N = R_{Th}$ the procedure and value obtained using the approach described for Thevinin's theorem will determine the proper value of R_N.

Finding I_N :

Calculate I_N by first returning all the sources to their original position and then finding the short- circuit current between the marked terminals. It iis the same current that would be measured by an ammeter placed between the marked terminals.

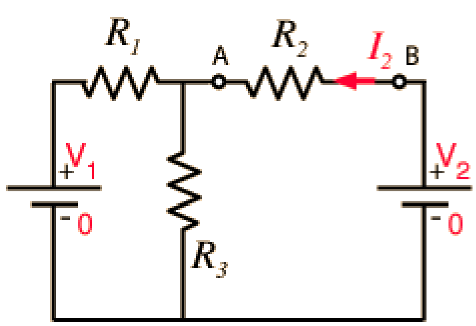
 Conclusion:

5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.



$$r = R_2 + \frac{R_1R_3}{R_1 + R_3}$$

NUMERICAL EXAMPLE APPLIED TO TWO LOOP CIRCUIT :

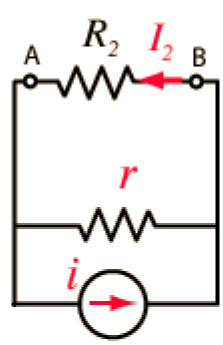


$$I_2 = \frac{ir}{(r + R_2)}$$

$$V_{AB} = V_2 - \frac{V_1R_3}{R_1 + R_3}$$

$$r = R_1 \parallel R_3 = \frac{R_1R_3}{R_1 + R_3}$$

$$i = \frac{V_{AB}}{r}$$



For R_1 = Ω, R_2 = Ω, R_3 = Ω,

and voltages V_1 = ☐ V and V_2 = ☐ V,

the open circuit voltage is $V_{AB} = V_2 - \dfrac{V_1 R_3}{R_1 + R_3}$ = ☐ V

since R_1 and R_3 form a simple voltage divider.

The Norton resistance is $r = R_1 \| R_3 = \dfrac{R_1 R_3}{R_1 + R_3}$ = ☐ Ω.

This gives a Norton current $i = \dfrac{V_{AB}}{r}$ = ☐ A.

Finally the calculated current is $I_2 = \dfrac{ir}{(r + R_2)}$ = ☐ A

Note: To avoid dealing with so many short circuits, any resistor with value zero will default to 1 when a voltage is changed. It can be changed back to a zero value if you wish to explore the effects of short circuits. Ohms and amperes are the default units, but if you put in resistor values in kilohms, then the currents will be milliamperes.

SUPERPOSITION THEOREM:

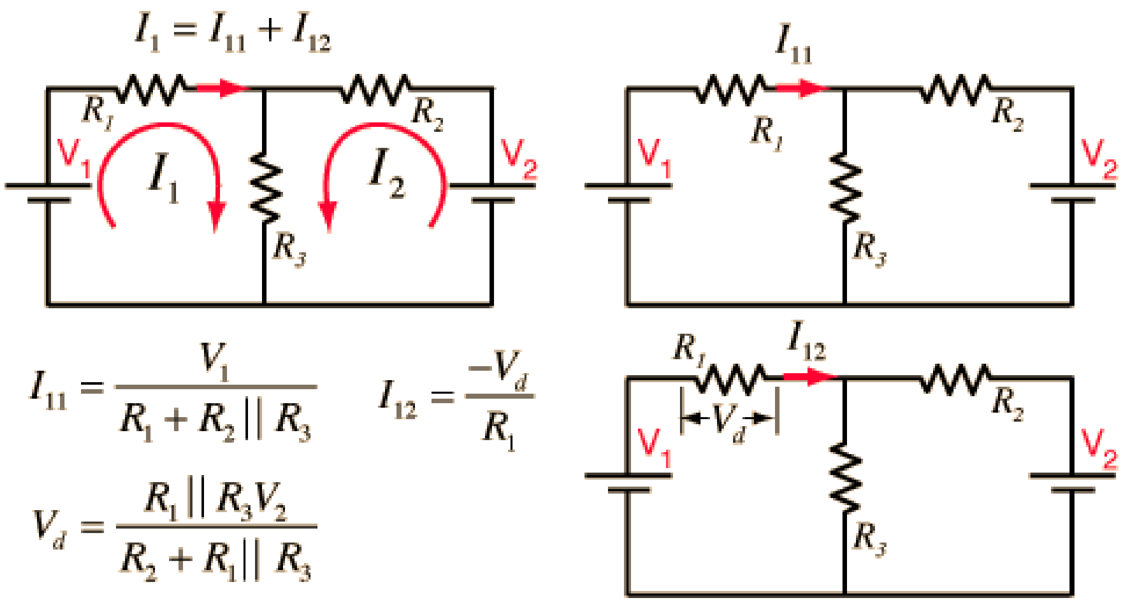
Used to find the solution to networks with two or more sources that are not in series or parallel.

The current through, or voltage across, an element in a network is equal to the algebraic sum of the currents or voltages produced independently by each source.

Since the effect of each source will be determined independently, the number of networks to be analyzed will equal the number of sources.

The total power delivered to a resistive element must be determined using the total current through or the total voltage across the element and cannot be determined by a simple sum of the power levels established by each source.

To apply the superposition theorem to calculate the current through resistor R_1 in the two loop circuit shown, the individual current supplied by each battery is calculated with the other battery replaced by a short circuit.



$$I_{11} = \frac{V_1}{R_1 + R_2 \,||\, R_3} \qquad I_{12} = \frac{-V_d}{R_1}$$

$$V_d = \frac{R_1 \,||\, R_3 V_2}{R_2 + R_1 \,||\, R_3}$$

$R_1 \,||\, R_3$ means the parallel resistance of R_1 and R_3.

For R_1 = ____ Ω, R_2 = ____ Ω, R_3 = ____ Ω,

and voltages V_1 = ____ V and V_2 = ____ V,

the calculated currents are

$$I_{11} = \frac{V_1}{R_1 + R_2 \,||\, R_3} = \text{____ A}, \quad I_{12} = \frac{-V_d}{R_1} = \text{____ A}$$

with a resultant current in R_1 of $I_1 = I_{11} + I_{12}$ = ____ A.

Note: To avoid dealing with so many short circuits, any resistor with value zero will default to 1 when a voltage is changed. It can be changed back to a zero value if you wish to explore the effects of short circuits. Ohms and amperes are the default units, but if you put in resistor values in kilohms, then the currents will be milliamperes.

MAXIMUM POWER TRANSFER THEOREM:

The maximum power transfer theorem states the following:

A load will receive maximum power from a network when its total resistive value is exactly equal to the Thévenin resistance of the network applied to the load.

That is, $R_L = R_{Th}$

For loads connected directly to a dc voltage supply, maximum power will be delivered to the load when the load resistance is equal to the internal resistance of the source; that is, when:

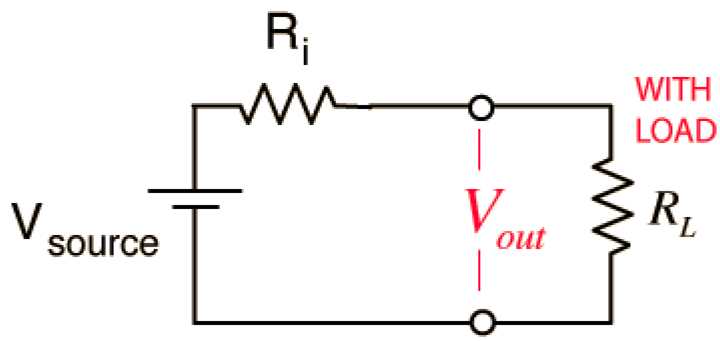
$$R_L = R_{int}$$

Any number of parallel voltage sources can be reduced to one.

This permits finding the current through or voltage across R_L without having to apply a method such as mesh analysis, nodal analysis, superposition and so on.

1. Convert all voltage sources to current sources.
2. Combine parallel current sources.
3. Convert the resulting current source to a voltage source and the desired single-source network is obtained.

As a general rule, the maximum power transfer from an active device like a power supply or battery to an external device occurs when the impedance of the external device matches that of the source. That optimum power is 50% of the total power when the impedance of the active device is matched to that of the load. Improper impedance matching can lead to excessive power use and possible component damage. This situation will be modeled here for strictly resistive impedances.



OUTPUT VOLTAGE UNDER LOAD

$$V_{out} = V_{source}\frac{R_L}{(R_i + R_L)}$$

4.

5.

For R_i = ☐ Ω, R_L = ☐ Ω, and V_{source} = ☐ V,

The open circuit output voltage would be equal to V_{source}, but when it is connected to the load, the output voltage will drop to V_{out} = ☐ V.

For this circuit, the total power supplied by the power supply is P_{total} = ☐ watts and the power delivered to the load resistor R_L is P_{out} = ☐ watts.

The load then receives ☐ % of the total power.

This is an important practical situation in DC circuits, enabling you to model the output of batteries with internal resistance and other situations where the power supply has internal resistance. Note that the power output from the voltage source, which is assumed to be ideal, is maximum when the load resistor R_L is equal to the internal resistance R_i, delivering 50% of the source power to the load. You can get a higher percentage of the power to the load by increasing the load resistor, and that is the desirable situation with a battery with low internal resistance. Any power used up in the internal resistor is lost to heat. But the absolute power to the load will be diminished.